What is the Noise Figure of a Matched Attenuator in Thermal Equilibrium?

First of all we have to think a bit about what an attenuator is and how it differs from, say, an amplifier. We are going to assume:

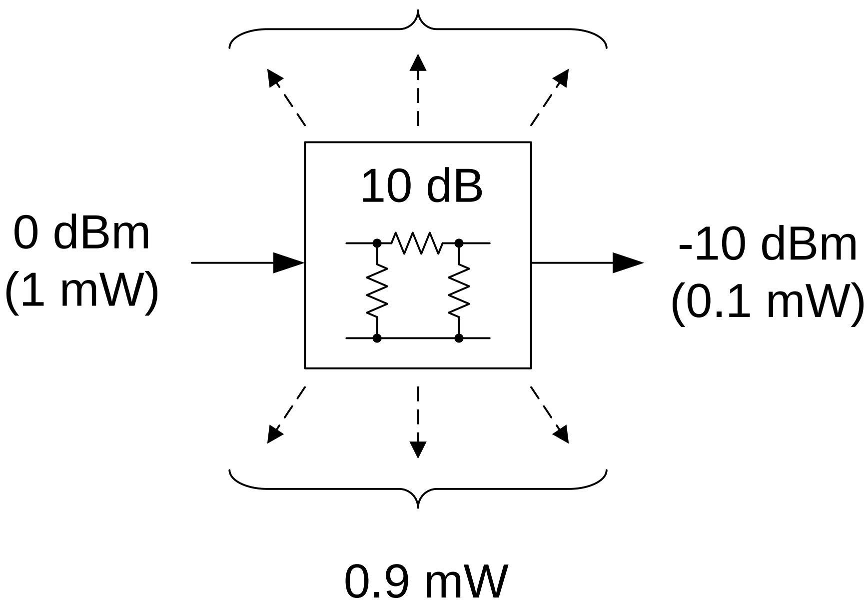
- The attenuator is perfectly matched at both its input and output.
- The attenuator is in thermal equilibrium with its surroundings.

The need for it to be perfectly matched means that we can ignore for now any reflected power caused by mismatches at the input or output. We will look at mismatched devices elsewhere.

By thermal equilibrium, we mean that there are no mechanisms for heat transfer into or out of the attenuator other than that resulting from the electrical signal or noise power dissipated in it. In practice that means that it is at the same temperature as its surroundings before any power is applied and there are no surfaces nearby that could participate in heat transfer.

A Typical Attenuator

Supposing we applied a 0 dBm signal to such a 10 dB matched attenuator as shown in the schematic. What happens to the power that goes into it (the incident power)?



As it is a 10 dB attenuator, the output power will be 10 dB less than the input power, or – 10 dBm as shown. To understand what happens to the power we need to convert the logarithmic (dBm) absolute power quantities into linear equivalents such as milliwatts (mW). The definition for a power level expressed in decibels relative to one milliwatt (dBm) or P_{dBm} is given by the following:

$$P_{dBm} = 10\log_{10}\left(\frac{P_{mW}}{1\,mW}\right) \quad dBm$$

where P_{mW} is the power in mW. Similarly, the conversion from dBm to mW is

$$P_{mW} = 10^{\frac{P_{dBm}}{10}} \quad mW$$

Using the last equation to convert the input and output powers into their linear values, we have 1 mW incident at the attenuator input and 0.1 mW leaving the attenuator at the other port. So what happens to the other 0.9 mW? It is dissipated in the attenuator in the form of heat. 0.9 mW is not very much power so it is not going to get very warm but the principle of course applies to any power levels that we use.

Attenuators must all contain elements of resistance in some shape or form, all of which dissipate heat. *Perfectly* reactive components such as inductors and capacitors do not dissipate any heat because, by definition, they are not resistors and are therefore not capable of dissipating heat.

Noise Figure Definition

Now we will look at the noise figure definition before applying it to a perfectly matched attenuator in thermal equilibrium. More precisely we will start with the definition of the noise factor, a linear quantity F. This is directly related to noise figure, the latter being more common in everyday use and on datasheets. The noise figure, represented by the lower case letter f is simply the logarithmic equivalent of noise factor and expressed in dB, so

$$f = 10\log_{10} F \quad dB$$

The noise factor of a 2-port matched device in thermal equilibrium, including an attenuator, is a unit less quantity defined as the linear ratio of the input signal to noise ratio (SNR) to the output SNR [2][5]. Therefore

$$F = \frac{S_i / N_i}{S_o / N_o} = \frac{N_o}{GN_i}$$

Where

S_i is the input signal power;

N_i is the input noise power;

S_o is the output signal power;

N_o is the output noise power;

G is the linear gain of the 2-port network.

All of there parameters must of course be expressed in the same linear units such as microwatts (µW) or milliwatts (mW). The linear gain quantity is unit less.

Excess Noise Power

A noisy device is one which adds some level of thermal noise to a signal that is propagated through it. Such a device would have a noise *factor* which is greater than unity, or alternatively a noise *figure* greater than 0 dB. In a practical communications system there will also be some level of signal and noise presented at the input of the device. In the last section, we denoted these by S_i and N_i respectively. At the output of the noisy device there will be two uncorrelated components of noise:

- that due to the input noise only but linearly amplified, just as a signal is amplified;
- the noise added by the device itself, or excess noise power N_x, referred to the output.

If we were using continuous waves we would need to know about their frequencies and phase relationship, if any. With thermal noise, if they are uncorrelated we simply add their powers at the same point.

Putting that mathematically, we have [5]:

$$N_0 = GN_i + N_x$$

Substituting for N_0 in the noise factor definition equation [5],

$$F = \frac{N_o}{GN_i} = \frac{GN_i + N_x}{GN_i} = 1 + \frac{N_x}{GN_i}$$

Noise Power From a Perfectly Matched Thermal Noise Source

The noise factor definition requires some level of thermal noise to be applied at the input of the noisy device. From the theory of blackbody radiation with some approximations introduced by Rayleigh and Jeans, the noise power P_N from a perfectly matched thermal noise source is given by [3]:

$$P_N = kTB$$

where

$k = 1.38 \times 10^{-23}\ J/K$ is Boltzmann's constant;

T is the absolute noise temperature, expressed in Kelvin (K);

B is the noise bandwidth under consideration, or resolution bandwidth (Hz).

Note that B is not related to the bandwidth of the device itself.

Therefore, in the case of the noise factor definition given above, the input thermal noise power is N_i, so, in this case,

$$N_i = kTB$$

and therefore [5]

$$F = \frac{N_o}{GN_i} = \frac{GN_i + N_x}{GN_i} = 1 + \frac{N_x}{GkTB}$$

This shows that, for a fixed resolution bandwidth, the noise factor is actually dependent on the noise temperature of the source connected to the input of the device.

Standard Noise Temperature

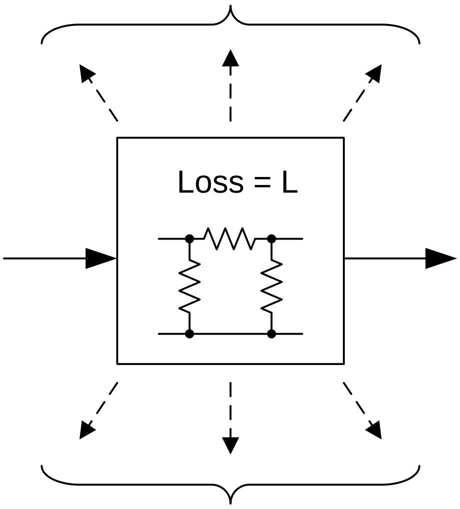
We can see from the noise factor definition equation that the noise factor F is a function of the input noise power N_i. The noise factor of a given device in a system will therefore depend on the actual input noise power it receives in the same system. In order to define a standard way of expressing noise factors of devices on datasheets, it is very common to define a standard value for noise factor as that with a perfectly matched thermal noise source at a temperature T_0 connected to the input, where $T_0 = 290\ K$. This was standardized by the Institution of Radio Engineers (IRE), a predecessor of the Institution of Electrical and Electronics Engineers (IEEE).

In the standard temperature definition of noise factor therefore T_0 replaces T so [5],

$$F = \frac{N_o}{GN_i} = \frac{GN_i + N_x}{GN_i} = 1 + \frac{N_x}{GkT_0B}$$

The Particular Case of a Matched Attenuator in Thermal Equilibrium

The case of the noise figure definition applied to the perfectly matched attenuator is shown schematically in the following diagram [1].



$$\frac{N_i}{B} = kT_0 \qquad \frac{N_o}{B} = \frac{N_i}{BL} = \frac{kT_0}{L}$$

$$\frac{N_x}{B} = \frac{N_i}{B} - \frac{N_o}{B}$$
$$= \frac{N_i}{B}\left(1 - \frac{1}{L}\right)$$
$$= kT_0\left(1 - \frac{1}{L}\right)$$

It is useful to define the linear loss factor (L) for the attenuator under consideration. This is simply the ratio of the input power to the output power or the reciprocal of its linear gain, so

$$L = \frac{S_i}{S_o} = \frac{1}{G}$$

Often the loss of an attenuator is expressed in the logarithmic decibel (dB) unit. Since we have qualified the property as a 'loss' and not a gain, the loss is always a positive value in dB for attenuators. If the linear loss factor of an attenuator is L , its logarithmic loss, $l\ dB$ is given by

$$l = 10\log_{10} L \quad dB$$

Again, in order to define its noise factor, a perfectly matched thermal noise source of noise temperature T_0 is connected to the input. The input noise power N_i is therefore given by:

$$N_i = kT_0 B$$

We have not said much about the resolution bandwidth B . In fact it does not matter what it is as long as it does not change once we have decided on a value. It is very common to choose $B = 1\,Hz$ simply because it makes the arithmetic easier when it comes to making substitutions. There is no other reason, we could have chosen any value for B .

With the thermal noise source at temperature T_0 connected to the input of the attenuator, the input noise power density (NPD) is

$$\frac{N_i}{B} = kT_0$$

The output NPD caused by the action of the attenuator in exactly the same way as it would have attenuated a signal, is

$$\frac{N_o}{B} = \frac{N_i}{BL} = \frac{kT_0}{L}$$

So what happened to the rest of the noise power, in fact some 90% of it in the case of a 10 dB attenuator? It was dissipated in the form of heat in the attenuator itself. Again, in exactly the same way as a signal would have been affected. In this case a very small amount of heat, but definitely heat. As we have already declared that the attenuator is in thermal equilibrium, there are no heat flows on account of any temperature gradients, only that due to the noise power itself. The heat dissipated in the attenuator is therefore equivalent to the excess noise power of the attenuator. The thermal noise power entering the attenuator was split into two parts, the attenuated part and the excess noise part. Therefore the NPD of the excess noise power is

$$\begin{aligned}\frac{N_x}{B} &= \frac{N_i}{B} - \frac{N_o}{B} \\ &= \frac{N_i}{B}\left(1 - \frac{1}{L}\right) \\ &= kT_0\left(1 - \frac{1}{L}\right)\end{aligned}$$

Now we can substitute this value into the noise factor definition as follows:

$$\begin{aligned}F &= 1 + \frac{N_x}{GkT_0B} \\ &= 1 + \frac{L}{kT_0}\frac{N_x}{B} \\ &= 1 + L\left(1 - \frac{1}{L}\right) \\ &= L\end{aligned}$$

So the linear loss factor of a perfectly matched attenuator in thermal equilibrium is equivalent to its noise factor. Therefore, under the same conditions, the logarithmic loss of an attenuator is equivalent to its noise figure.

So intuitively perhaps the thermal noise performance of an attenuator is easier to understand than that of an amplifier. An attenuator does not receive power from any source other than the signal and noise applied to the input. There is a simple relationship between the input and output powers and the power remaining is the excess noise. Then the noise factor definition may be applied.

References

1. Minkoff, John; *Signal Processing Fundamentals and Applications for Communications and Sensing Systems*; Artech House, Inc.; pp 59 – 60. ISBN 1-58053-360-4 (2002).
2. Pozar, David M.; *Microwave Engineering – Third Edition*; John Wiley & Sons Inc.; p 493; ISBN 0-471-44878-8 (2005).
3. Pozar (op. cit.): pp 489 -490.
4. Pozar (op. cit.): pp 497 -499.
5. Agilent; *Fundamentals of RF and Microwave Noise Figure Measurements*; Application Note 57-1; Literature Number 5952-3706; pp 7 - 8.