1 SCATTERING PARAMETERS

1.1 S and T Parameters

Scattering parameters or S-parameters are terminology used in electrical engineering and communication systems to describe the electrical behavior of linear electrical networks when undergoing various steady state stimulii by small signals. They are members of a family of parameters used in electronics engineering, other examples being: Y-parameters,[1] Z-parameters,[2] H-parameters, T-parameters and ABCD-parameters.[3][4] Although applicable at all frequencies, S-parameters are mostly measured and specified for networks operating at RF and microwave frequencies as they represent parameters particularly useful at RF. In general, S-parameters change with the measurement frequency so this must be included for any S-parameter measurements stated. S-parameters are readily represented in matrix form and therefore obey the rules of matrix algebra. Many useful electrical properties of networks or components may be expressed using S-parameters such as gain, return loss, voltage standing wave ratio (VSWR) and amplifier stability. The term 'scattering' is perhaps more common to optical engineering, referring to the effect observed when a plane electromagnetic wave is incident on an obstruction or passes across a boundary of dissimilar dielectric media. In the context of S-parameters, scattering refers to the way in which the travelling currents and voltages in a transmission line are affected when they meet a discontinuity, an electrical impedance differing from the line's characteristic impedance.

1.1.1 Background

Historically, an electrical network would have comprised a 'black box' containing various interconnected basic electrical lumped elements (resistors, capacitors and inductors), but here it is normally understood that it may contain any components which behave in a linear way such as microstrip lines, ferrite components, transformers etc.. It is also extended to include many typical communication systems components or 'blocks' such as amplifiers, attenuators, filters, couplers and equalizers provided they are also operating under linear and defined conditions.

An electrical network to be described by S-parameters may have any number of ports. Ports are the points at which electrical currents either enter or exit the network. Sometimes these are referred to as pairs of 'terminals'[5][6] so, for example, a 2-port network is equivalent to a 4-terminal network, though this terminology is less common with S-parameters, because most S-parameter measurements are made at frequencies where coaxial connectors are more appropriate.

The S-parameter matrix describing an N-port network will be square of dimension 'N' and will therefore contain N^2 elements. At the test frequency each element or S-parameter is represented by a complex number, thus providing magnitude or amplitude and phase, expressed either in rectangular or, more commonly, in polar form. S-parameters do not have any units when they are expressed in linear form but in polar form the magnitude of the parameter will often be shown logarithmically or in 'log magnitude' typically in dB. Any such S-parameter may be displayed graphically on a polar diagram or, if it applies to one port only (being of the form S_{nn}), it may be displayed on a normalised impedance or normalised admittance Smith Chart. The Smith Chart allows simple conversion between the S_{nn} parameter, equivalent to the voltage reflection coefficient and the (normalised) impedance (or admittance) 'seen' at that port.

The following information must be defined when specifying an S-parameter matrix:

- The nominal system impedance (for probably 90% of the cases this is 50 Ω
- The allocation of port numbers.
- Dependent conditions such as frequency, temperature, control voltage, bias current etc., where applicable.

1.1.2 The General S-Parameter Matrix

For a generic multi-port network, it is assumed that all ports except the one or pair under consideration are terminated in loads identical to the system impedance and each of the ports is allocated a number 'n' ranging from 1 to N, where N is the total number of ports. For port n, the associated S-parameter definition is in terms of incident and reflected 'power waves', a_n and b_n respectively. These are normalised versions of the corresponding incident and reflected travelling voltage waves, V_n^+ and V_n^- respectively, according to transmission line theory. They are related in terms of the system impedance Z_0 thus:

$$a_n = \frac{V_n^+}{\sqrt{Z_0}}$$

and

$$b_n = \frac{V_n^-}{\sqrt{Z_0}}$$

For all ports of the entire network, the reflected power waves may be defined in terms of the S-parameter matrix and the incident power waves by the following matrix equation:

$$\begin{pmatrix} b_1 \\ . \\ . \\ b_n \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} & . & S_{1n} \\ S_{21} & . & . & . \\ . & . & . & . \\ S_{n1} & . & . & S_{nn} \end{pmatrix} \begin{pmatrix} a_1 \\ . \\ . \\ a_n \end{pmatrix}$$

S-parameters are individually represented by the upper case letter 'S' followed by two integer subscripts indicating in order, the row and the column of the position of the S-parameter in the S-parameter matrix. All S-parameters are complex quantities and therefore represent the linear magnitude and "spatial" phase at the test frequency. S-parameters do not represent the temporal (time-related) phase.

1.1.3 Two-Port Networks

The S-parameter matrix for the 2-port network is probably the most common and it serves as the basic building block for the higher order matrices. In this case the relationships between the reflected, incident power waves and the S-parameter matrix is given by:

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

Expanding the matrices into equations gives:

$$b_1 = S_{11} a_1 + S_{12} a_2$$

$$b_2 = S_{21} a_1 + S_{22} a_2$$

Each equation gives the relationship between the reflected and incident power waves at each of the networks ports, 1 and 2, in terms of the network's individual S-parameters, S_{11}, S_{12}, S_{21} and S_{22}. If one considers an incident power wave at port 1 (a_1) there may result from it waves exiting from either port 1 itself (b_1) or port 2 (b_2). However if, according to the definition of S-parameters, port 2 is terminated in a load identical to the system impedance (Z_0) then, by the maximum power transfer theorem, b_2 will be totally absorbed making a_2 equal to zero. Therefore

$$S_{11} = \frac{b_1}{a_1} = \frac{V_1^-}{V_1^+} \text{ and } S_{21} = \frac{b_2}{a_1} = \frac{V_2^-}{V_1^+}$$

Similarly, if port 1 is terminated in the system impedance then a_1 becomes zero, giving

$$S_{12} = \frac{b_1}{a_2} = \frac{V_1^-}{V_2^+} \text{ and } S_{22} = \frac{b_2}{a_2} = \frac{V_2^-}{V_2^+}$$

Each 2-port S-parameter has the following generic descriptions:

S_{11} is the input port voltage reflection coefficient

S_{12} is the reverse voltage gain

S_{21} is the forward voltage gain

S_{22} is the output port voltage reflection coefficient

1.1.4 Reciprocity

A network will be reciprocal if it is passive and it contains only isotropic materials. For example, attenuators, cables, splitters and combiners are all reciprocal networks and $S_{mn} = S_{nm}$ in each case. The S-parameter matrix will be equal to its transpose. All networks which include anisotropic materials such as those containing ferrite components will be non-reciprocal. Although it doesn't necessarily contain ferrites, an amplifier is also an example of a non-reciprocal network.

An interesting property of 3-port networks however is that they cannot be simultaneously reciprocal, loss-free and perfectly matched at the same time.[7]

1.1.5 The Loss Free Network

A loss-free network is one which does not dissipate any power, or

$$\sum |a_n|^2 = \sum |b_n|^2 .$$

The sum of the incident powers at all ports is equal to the sum of the reflected powers at all ports. This implies that the S-parameter matrix is unitary, or

$$(S)(S)^* - (I) = 0$$

where S* is the complex conjugate of the transpose of (S) and (I) is the identity matrix.

1.1.6 Lossy Networks

A lossy network is one in which the sum of the incident powers at all ports is greater than the sum of the reflected powers at all ports and therefore dissipates power, or

$$\sum |a_n|^2 \neq \sum |b_n|^2$$

In this case

$$\sum |a_n|^2 \succ \sum |b_n|^2$$

and

$$(S)(S)^* - (I) \succ 0$$

1.1.7 Common Properties of 2-Port Devices Expressed in Terms of S-Parameters

An amplifer operating under linear (small signal) conditions is a good example of a non-reciprocal network and a matched attenuator is an example of a reciprocal network. In the following cases we will assume that the input and output connections are to ports 1 and 2 respectively. The nominal system impedance, frequency and any other factors which may influence the device, such as temperature, must also be specified.

1.1.7.1 Complex Linear Gain

The complex linear gain, G is given by

$$G = S_{21}$$

That is simply the complex voltage gain as a linear ratio of the complex output voltage divided by the complex input voltage, all values expressed in complex quantities.

1.1.7.2 Scalar Linear Gain

The scalar linear gain (or linear gain magnitude) |G| is given by

$$|G| = |S_{21}|$$

That is simply the voltage gain as a linear ratio of the output voltage divided by the input voltage. As this is a scalar quantity, the phase is not relevant in this case.

1.1.7.3 Scalar Logarithmic Gain

The scalar logarithmic (decibel or dB) expression for gain (g) is

$$g = 20\log_{10}|S_{21}| \quad dB$$

This is more commonly used than scalar linear gain and a positive quantity is understood as simply a 'gain'. A negative quantity can be expressed as a 'negative gain' or more usually as a 'loss' equivalent to its magnitude in dB. For example, a 10 m length of cable may have a gain of - 1 dB at 100 MHz or a loss of 1 dB at 100 MHz.

1.1.7.4 Input Return Loss

Input return loss (RL_{in}) is a measure of how close the actual input impedance of the network is to the nominal system impedance value and, expressed in logarithmic magnitude, is given by

$$RL_{in} = \left|20\log_{10}|S_{11}|\right| \quad dB$$

By definition, return loss is a positive scalar quantity implying the 2 pairs of magnitude (|) symbols. The linear part, ($|S_{11}|$) is equivalent to the reflected voltage magnitude divided by the incident voltage magnitude.

1.1.7.5 Output Return Loss

The output return loss (RL_{out}) has a similar definition to the input return loss but applies to the output port (port 2) instead of the input port. It is given by

$$RL_{in} = \left|20\log_{10}|S_{22}|\right| \quad dB$$

1.1.7.6 Reverse Gain and Reverse Isolation

The scalar logarithmic (decibel or dB) expression for reverse gain (**g_{rev}</math>) is:

$$g_{rev} = 20\log_{10}|S_{12}| \quad dB$$

Often this will be expressed as reverse isolation (I_{rev}) in which case it becomes a positive quantity equal to the magnitude of g_{rev} and the expression becomes:

$$I_{rev} = |g_{rev}| = \left|20\log_{10}|S_{12}|\right| \quad dB$$

1.1.7.7 Complex Voltage Reflection Coefficient

The complex voltage reflection coefficient at the input port (ρ_{in}) or at the output port (ρ_{out}) are equivalent to S_{11} and S_{22} respectively, so

$\rho_{in} = S_{11}$ and $\rho_{out} = S_{22}$

As S_{11} and S_{22} are complex quantities, so are ρ_{in} and ρ_{out}. The complex voltage reflection coefficient is often simply referred to as reflection coefficient.

1.1.7.8 Voltage Standing Wave Ratio

The voltage standing wave ratio (VSWR) at a port, represented by the lower case 's', is a similar measure of port match to return loss but is a scalar linear quantity, the ratio of the standing wave maximum voltage to the standing wave minimum voltage. It therefore relates to the magnitude of the voltage reflection coefficient and hence to the magnitude of either S_{11} for the input port or S_{22} for the output port.

At the input port, the VSWR (s_{in}) is given by

$$s_{in} = \frac{1+|S_{11}|}{1-|S_{11}|}$$

At the output port, the VSWR (s_{out}) is given by

$$s_{out} = \frac{1+|S_{22}|}{1-|S_{22}|}$$

1.1.8 S-Parameters in Amplifier Design

The reverse isolation parameter S_{12} determines the level of feedback from the output of an amplifier to the input and therefore influences its stability (its tendancy to refrain from oscillation). An amplifier with perfectly isolated input and output ports would have infinite scalar log magnitude isolation or the linear magnitude of

S_{12} would be zero. Such an amplifier is said to be unilaterial or a perfect 'buffer' (the power only flows in one direction). Most practical amplifers though will have some finite isolation allowing the reflection coefficient 'seen' at the input to be influenced to some extent by the load connected on the output.

Suppose the output port of a real (non-unilaterial or bilaterial) amplifier is connected to an arbitrary load which exhibits a reflection coefficient of ρ_L. The actual reflection coefficient 'seen' at the input port (ρ_{IN}) will be given by:[8]

$$\rho_{IN} = S_{11} + \frac{S_{12}S_{21}\rho_L}{1 - S_{22}\rho_L}$$

If the amplifier is unilaterial then $|S_{12}| = 0$ and $\rho_{IN} = S_{11}$ or, to put it another way, the output loading has no effect on the input.

A similar property exists in the opposite direction, in this case if ρ_{OUT} is the reflection coefficient seen at the output port and ρ_S is the reflection coeffficient of the source connected to the input port.

$$\rho_{IN} = S_{22} + \frac{S_{12}S_{21}\rho_S}{1 - S_{11}\rho_S}$$

1.1.8.1 The Port Loading Conditions for an Amplifier to be Unconditionally Stable

An amplifier is unconditionally stable if a load or source of "any" reflection coefficient can be connected without causing instability. This condition occurs if the magnitudes of the reflection coefficients at the source, load and the amplifier's input and output ports are all less than unity across an appreciably greater frequency range than the normal operating frequency range. Instability can cause severe distortion of the amplifier's gain frequency response or, in the extreme, oscillation. To be unconditionally stable at the frequency of interest, an amplifier must satisify the following 4 equations simultaneously:[9]

$$|\rho_S| \le 1$$

$$|\rho_L| \le 1$$

$$|\rho_{IN}| \le 1$$

$$|\rho_{OUT}| \le 1$$

The boundary condition for when each of these values is equal to unity may be represented by a circle drawn on the polar diagram representing the (complex) reflection coefficient, one for the input port and the other for the output port. Often these will be scaled as Smith Charts. In each case coordinates of the circle centre and the associated radius are given by the following equations:

ρ_L Values for $|\rho_{IN}|$=1 (Output Stability Circle)

$$Radius \quad r_L = \left| \frac{S_{12}S_{21}}{|S_{22}|^2 - |\Delta|^2} \right|$$

$$Centre \quad c_L = \frac{(S_{22} - \Delta S_{11}{*})^*}{|S_{22}|^2 - |\Delta|^2}$$

ρ_S Values for $|\rho_{OUT}|$=1 (Input Stability Circle)

$$Radius \quad r_S = \left| \frac{S_{12}S_{21}}{|S_{11}|^2 - |\Delta|^2} \right|$$

$$Centre \quad c_L = \frac{(S_{11} - \Delta S_{22}{*})^*}{|S_{11}|^2 - |\Delta|^2}$$

where, in both cases

$$\Delta = S_{11}S_{22} - S_{12}S_{21}$$

and the superscript star (*) indicates a complex conjugate.

The circles are in complex units of reflection coefficient so may be drawn on impedance or admittance based Smith Charts normalised to the system impedance. This serves to readily show the regions of normalised impedance (or admittance) for predicted unconditional stability. Another way of demonstrating unconditional stability is by means of the Rollet stability factor (K), defined as

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2\,|S_{12}S_{21}|}$$

The condition of unconditional stability is achieved when K>1 and |Δ|<1.

1.1.9 Scattering Transfer Parameters

The Scattering Transfer parameters or T-parameters of a 2-port network are expressed by the T-parameter matrix and are closely related to the corresponding S-parameter matrix. The T-parameter matrix is related to the incident and reflected normalised waves at each of the ports as follows:

$$\begin{pmatrix} b_1 \\ a_1 \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}$$

The advantage of T-parameters compared to S-parameters is that they may be used to readily determine the effect of cascading 2 or more 2-port networks by simply multiplying the associated individual T-parameter matrices. If the T-parameters of say three different 2-port networks 1, 2 and 3 are (T_1), (T_2) and (T_3) respectively then the T-parameter matrix for the cascade of all three networks (T_T) in serial order is given by:

$(T_T)=(T_1)(T_2)(T_3)$

As with S-parameters, T-parameters are complex values and there is a direct conversion between the two types. Although the cascaded T-parameters is a simple matrix multiplication of the individual T-parameters, the conversion for each network's S-parameters to the corresponding T-parameters and the conversion of the cascaded T-parameters back to the equivalent cascaded S-parameters, which are usually required, is not trivial. However once the operation is completed, the complex full wave interactions between all ports in both directions will be taken into account. The following equations will provide conversion between S and T parameters for 2-port networks.[10]

From S to T:

$$T_{11} = \frac{-\det(S)}{S_{21}}$$

$$T_{12} = \frac{S_{11}}{S_{21}}$$

$$T_{21} = \frac{-S_{22}}{S_{21}}$$

$$T_{22} = \frac{1}{S_{21}}$$

From T to S

$$S_{11} = \frac{T_{12}}{T_{22}}$$

$$S_{12} = \frac{\det(T)}{T_{22}}$$

$$S_{21} = \frac{1}{T_{22}}$$

$$S_{22} = \frac{-T_{21}}{T_{22}}$$

1.1.10 One Port S-Parameters

The S-parameter for a 1-port network is given by a simple 1 x 1 matrix of the form (S_{nn}) where n is the allocated port number. To comply with the S-parameter definition of linearity, this would be a passive load of some type but may contain reactive elements.

1.1.11 Building Up Higher Order S-Parameter Matrices Using 2-Ports

Higher order S-parameters for pairs of dissimilar ports (S_{mn} where $m \neq n$) may be deduced similarly to those for 2-port networks by considering pairs of ports in turn, in each case ensuring that all of the remaining (unused) ports are loaded with an impedance identical to the system impedance. In this way the incident power wave for each of the unused ports becomes zero yielding similar expressions to those obtained for the 2-port case. S-parameters relating to single ports only (S_{mm}) require "all" of the remaining ports to be loaded with an impedance identical to the system impedance therefore making all of the incident power waves zero "except" that for the port under consideration. In general therefore we have:

$$S_{mn} = \frac{b_m}{a_n}$$

and

$$S_{mm} = \frac{b_m}{a_m}$$

For example, a 3-port network such as a 2-way splitter would have the following S-parameter definitions

$$S_{11} = \frac{b_1}{a_1} = \frac{V_1^-}{V_1^+}$$

$$S_{33} = \frac{b_3}{a_3} = \frac{V_3^-}{V_3^+}$$

$$S_{32} = \frac{b_3}{a_2} = \frac{V_3^-}{V_2^+}$$

$$S_{23} = \frac{b_2}{a_3} = \frac{V_2^-}{V_3^+}$$

1.1.12 The Measurement of S-Parameters

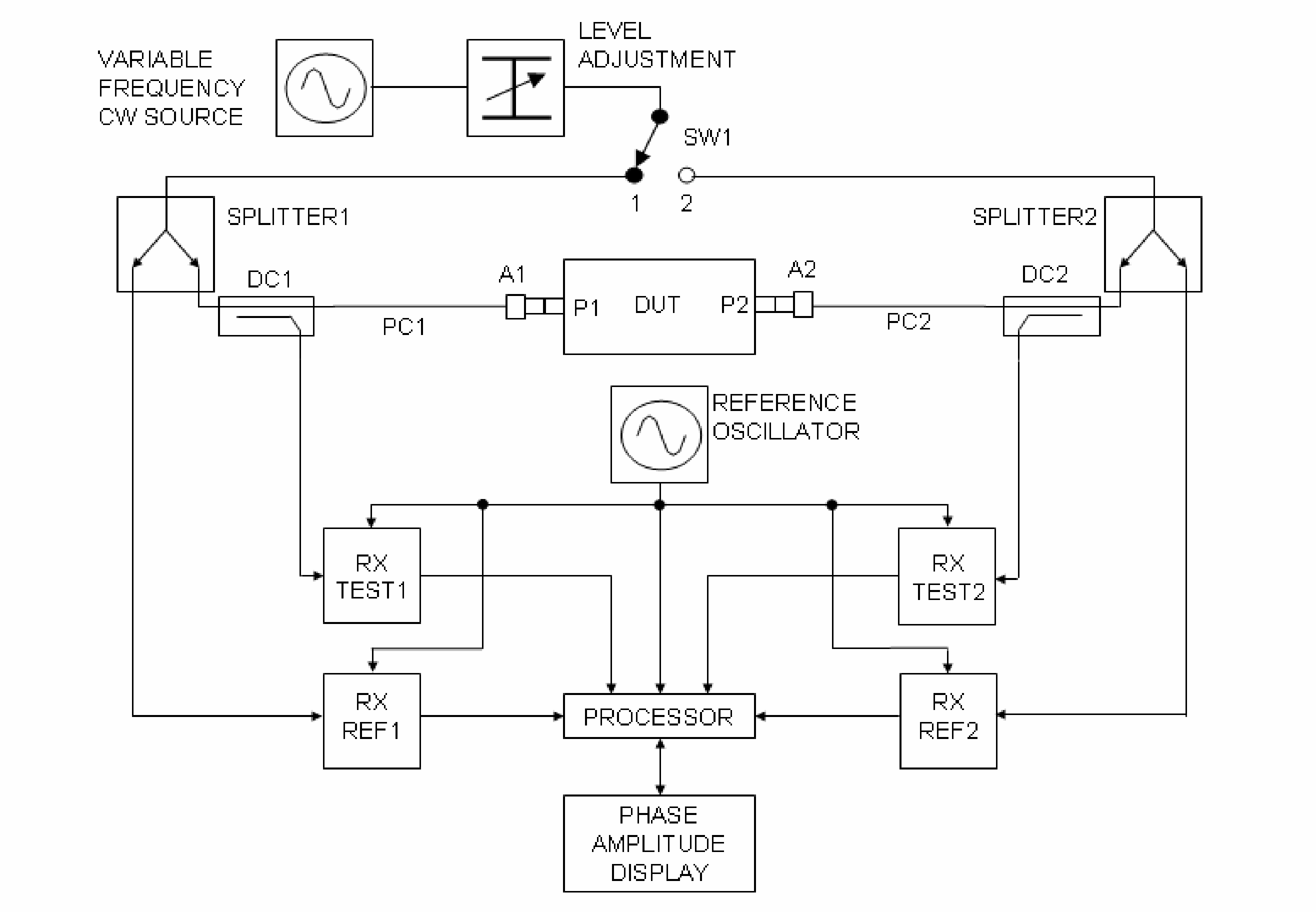

Figure 1-1 The basic system architecture of a two port vector network analyzer

1.1.12.1 The Vector Network Analyzer

Figure 1-1 shows the essential parts of a typical 2-port vector network analyzer (VNA). The two ports of the device under test (DUT) are denoted port 1 (P1) and port 2 (P2). The test port connectors provided on the VNA are precision types which will normally have to be extended and connected to P1 and P2 using precision cables 1 and 2, PC1 and PC2 respectively and suitable connector adaptors A1 and A2 respectively.

The test frequency is generated by a variable frequency CW source and its power level is set using a variable attenuator. The position of switch SW1 sets the direction that the test signal passes through the DUT. Initially consider that SW1 is at position 1 so the test signal is incident on the DUT at P1 which is appropriate for measuring S_{11} and S_{21}. The test signal is fed to the common port of splitter 1, one arm (the reference channel) feeding a reference receiver for P1 (RX REF1) and the other (the test channel) connecting to P1 via the [[directional coupler]] DC1, PC1 and A1. The third port of DC1 couples off the power reflected from P1 via A1 and PC1, then feeding it to test receiver 1 (RX TEST1). Similarly, signals leaving P2 pass via A2, PC2 and DC2 to RX TEST2. RX REF1, RX TEST1, RX REF2 and RXTEST2 are known as coherent receivers as they share the same reference oscillator, and they are capable of measuring both the test signal's amplitude and phase at the test frequency. All the complex receiver output signals are fed to a processor which does the mathematical processing and displays the chosen parameters and format on the phase and amplitude display. The instantaneous value of phase includes both the temporal and spatial parts, but the former is removed by virtue of using 2 test channels, one as a reference and the other for measurement. When SW1 is set to position 2, the test signals are applied to P2, the reference is measured by RX REF2, reflections from P2 are coupled off by DC2 and measured by RX TEST2 and signals leaving P1 are coupled off by DC1 and measured by RX TEST1.

1.1.12.2 Calibration

Initially, accurate calibration appropriate to the intended measurements is the essential. Several types of calibration are normally available on the VNA. It is only in the last few years that VNAs have had the sufficiently advanced processing capability required to accomplish the more advanced types of calibration, including corrections for systematic errors.[11] The more basic types, such as response type calibrations, may be performed quickly but will only provide a result with moderate uncertainty. For improved uncertainty and dynamic range a full 2 port calibration is required prior to DUT measurement. This will effectively eliminate all sources of systematic errors inherent in the VNA measurement system.

Systematic errors do not vary with time during a calibration. For a set of 2 port S-parameter measurements there are a total of 12 types of systematic errors which are measured and removed mathematically as part of the full 2 port calibration procedure. They are:

- directivity and crosstalk
- source and load mismatches
- frequency response errors caused by reflection and transmission tracking within the test receivers.

The calibration procedure requires initially setting up the VNA with all the cables, adaptors and connectors necessary to connect to the DUT but not at this stage connecting it. A calibration kit is used according to the connector types fitted to the DUT. This will normally include adaptors, nominal short circuits (SCs), open circuits (OCs) and load termination (TERM) standards of both sexes appropriate to the VNA and DUT connectors. Even with standards of high quality, when performing tests at the higher frequencies various stray capacitances and inductances will become apparent and cause uncertainty during the calibration. Data relating to the strays of the calibration kit are measured at the factory traceable to national standards and the results are programmed into the VNA memory prior to performing the calibration.

The calibration procedure is normally software controlled, and instructs the operator to fit various calibration standards to the ends of the DUT connecting cables as well as making a through connection. At each step the VNA processor captures data across the test frequency range and stores it. At the end of the calibration procedure, the processor uses the stored data thus obtained to apply the systematic error corrections to all subsequent measurements made. At this point the DUT is connected and a corrected measurement of its S-parameters made.

1.1.12.3 The Output Format of the Measured and Corrected S-Parameter Data

The S-parameter test data may be provided in many alternative formats, for example: list, graphical (Smith Chart) and graphical (polar diagram).

List Format

In list format the measured and corrected S-parameters are tabulated against frequency. The most common list format is known as Touchstone or SNP, where N is the number of ports. Commonly text files containing this information would have the filename extension '.s2p'. An example of SNP format listing for the full 2-port S-parameter data obtained for a device is shown in Figure 1-2.

```
! Created Fri Jul 21 14:28:50 2005
# MHZ S DB R 50
! SP1.SP
50   -15.4   100.2   10.2   173.5   -30.1   9.6   -13.4   57.2
51   -15.8   103.2   10.7   177.4   -33.1   9.6   -12.4   63.4
52   -15.9   105.5   11.2   179.1   -35.7   9.6   -14.4   66.9
53   -16.4   107.0   10.5   183.1   -36.6   9.6   -14.7   70.3
54   -16.6   109.3   10.6   187.8   -38.1   9.6   -15.3   71.4
```

Figure 1-2 Typical Touchstone or SnP S-parameter output format

Rows beginning with an exclamation mark contains only comments. The row beginning with the hash symbol indicates that in this case frequencies are in megahertz (MHZ), S-parameters are listed (S), magnitudes are in dB (DB) and the system impedance is 50 Ω (R 50). There are 9 columns of data. Column 1 is the test frequency in megahertz in this case. Columns 2, 4, 6 and 8 are the magnitudes of S_{11}, S_{21}, S_{12} and S_{22} respectively in dB and columns 3, 5, 7 and 9 are the angles of S_{11}, S_{21}, S_{12} and S_{22} respectively in degrees.

Graphical (Smith Chart)

Any 2-port S-parameter may be displayed on a Smith Chart using polar co-ordinates, but the most meaningful would be S_{11} and S_{22} since either of these may be converted if required directly into an equivalent impedance (or admittance) using the characteristic Smith Chart normalised impedance (or normalised admittance) scaling appropriate to the system impedance.

Graphical (Polar Diagram)

Any 2-port S-parameter may be displayed on a polar diagram using polar co-ordinates. In either graphical format each S-parameter at a particular test frequency is displayed as a dot. If the measurement is a sweep across several frequencies a dot will appear for each.

1.1.13 Measuring the S-Parameters of a One-Port Network

The S-parameter matrix for a network with just one port will have just one element represented in the form **S_{nn}</math>, where n is the number allocated to the port. Most VNAs provide a simple one-port calibration capability for one port measurement to save time if that is all that is required.

1.1.14 Measuring The S-Parameters of Networks with More than 2 Ports

VNAs designed for the simultaneous measurement of the S-parameters of networks with more than two ports are feasible but quickly become prohibitively complex and expensive. Usually their purchase is not justified since the required measurements can be obtained using a standard two-port calibrated VNA but with extra measurements followed by the correct interpretation of the results obtained. The required S-parameter matrix can be assembled from successive two port measurements in stages, two ports at a time, on each occasion with the unused ports being terminated in high quality loads equal to the system impedance. One risk of this approach is that the return loss or VSWR of the loads themselves must be suitably specified, essentially to be the best possible match over the test frequency range, or to be as close as possible to a perfect 50 Ohms, or whatever the nominal system impedance is. For a network with many ports, there may be a temptation, on grounds of cost, to inadequately specify the VSWRs of the loads. Some analysis will be necessary to determine what the worst acceptable VSWR of the loads will be.

Assuming that the extra loads are specified adequately, if necessary, two or more of the S-parameter subscripts are modified from those relating to the VNA (1 and 2 in the case considered above) to those relating to the network under test (1 to N, if N is the total number of DUT ports). For example, if the DUT has 5 ports and a two port VNA is connected with VNA port 1 to DUT port 3 and VNA port 2 to DUT port 5, the measured VNA results S_{11}, S_{12}, S_{21} and S_{22} would be equivalent to S_{33}, S_{35}, S_{53} and S_{55} respectively, assuming that DUT ports 1, 2 and 4 were terminated in adequate 50 Ω loads .This would provide 4 of the necessary 25 S-parameters.

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