Do you really mean 'RMS power'?

Quite often people refer to root mean squared (RMS) power when they really mean mean power. Why?

Supposing we have a sinusoidal voltage against time waveform of peak value (or amplitude) 5 V, and a frequency of 50 Hz.

$Vpeak := 5$ V

$frequency := 50Hz$

The period is the reciprocal of the frequency, so

$period := \frac{1}{frequency} = 0.02$ s That is 20 milliseconds (20 ms)

Instead of a continuous function, this one is made up of many closely spaced discrete steps as they are easier to deal with in a practical example like this. Each step interval is defined by a variable called time_step. Lets take that as 0.25 ms. That gives us 80 steps per cycle which should be a close approximation to continuous.

$time\_step := 0.25 \cdot 10^{-3}$ time step of 0.25 ms

Choose a load resistance (Load_R) of 50 Ohm, this will be required when we look at power later.

$Load\_R := 50$ Ohm

We will look at a few cycles just to confirm the effect. We will use 400 time steps in total (tot_steps) giving us a total waveform of 100 ms or 0.1 s. As the period is 20 ms, that covers exactly 5 cycles. It is important to take a whole number of cycles.

$tot\_steps := 400$

nval is a 'range variable' to handle the discrete steps. That means, in this case, it can have a range of values from zero to tot_steps in increments of 1.

$nval := 0, 1 .. tot\_steps$

Now to define the range of time from zero to 0.1 s in steps of 0.25 ms, called time_s. Notice that the variable time_s has a subscript nval so it also represents a range of values according to the value of nval.

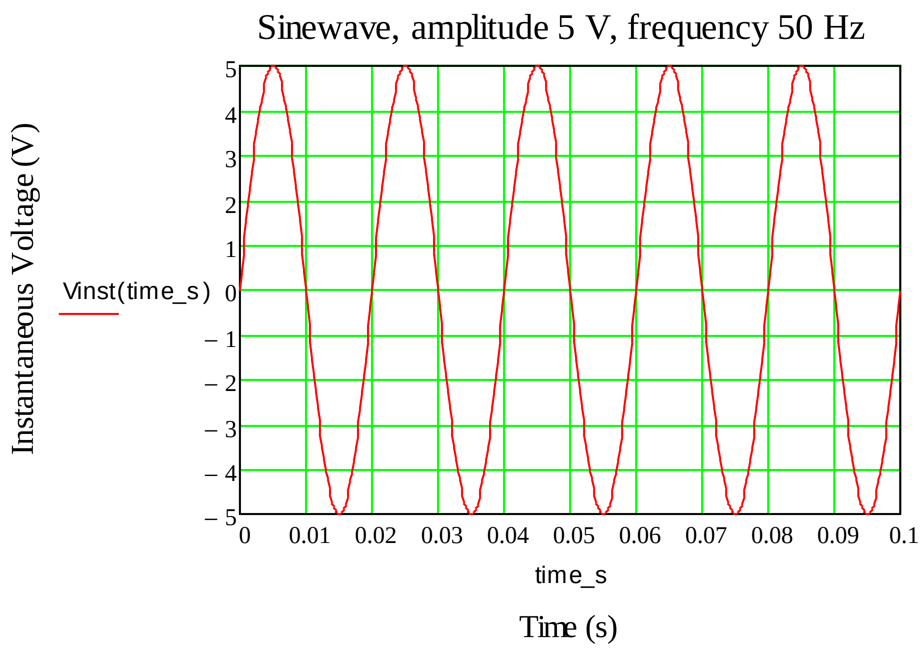
$time\_s_{nval} := nval \cdot time\_step$ s (range variable for the actual absolute time)

So when we use time_s without any subscript, we mean the whole set of time values defined by the equation above, from zero to 0.1 s.

This is the equation for the instantaneous voltage Vinst in volts (V) of the sinusoidal waveform covering the range from zero to 0.1 s. By instantaneous we mean 'the value at any instant', defined by the particular time at which we are looking at it.

$Vinst(time\_s) := Vpeak \cdot \sin\left(2 \cdot \pi \cdot \frac{time\_s}{period}\right)$ V instantaneous sinusoidal voltage

This is plotted below



Now to calculate the mean of that waveform, Vmean. Notice that it is being calculated over the complete waveform shown, 5 complete cycles. Usually, when we talk generally about the mean of a sinusoidal waveform we mean the 'long term mean' or over many cycles or a complete number of cycles, because of the periodic nature of the waveform. Here mval is an integer variable internal to the summation and used to work through each of the values through the whole waveform.

$$Vmean := \left(\frac{1}{tot\_steps}\right)\cdot\sum_{mval=0}^{tot\_steps} Vinst(mval\cdot time\_step) = 0 \qquad V$$

So its mean over a complete number of cycles is zero. That is because each positive half cycle cancels out each negative half cycle. We have 5 positive half cycles and 5 negative half cycles.

Next, the root mean square (RMS) value, VRMS. Slightly more complicated and notice that the instantaneous voltage values are squared so the negative ones are changed into positive values.

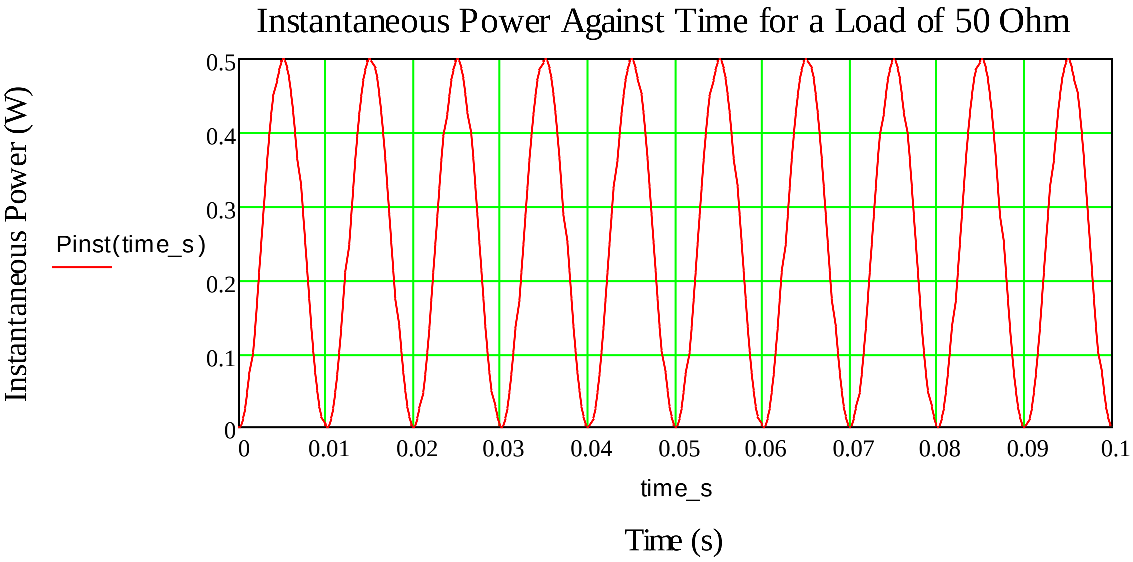
$$VRMS := \sqrt{\left(\frac{1}{tot\_steps}\right)\cdot\sum_{mval=0}^{tot\_steps} (Vinst(mval\cdot time\_step))^2} = 3.536 \qquad V$$

So the RMS value of this waveform is 3.536 V

To calculate the instantaneous power in watts (W) when the voltage waveform is applied across a perfect resistor of value Load_R in Ohms. The instantaneous voltage is squared and divided by the load resistance.

$$Pinst(time\_s) := \frac{(Vinst(time\_s))^2}{Load\_R} \qquad W \qquad \text{instantaneous power}$$

This is plotted below.



Some interesting things to note about the instantaneous power against time waveform:

It is positive only, we cannot have negative real power.

Its peak value is 0.5 W

The frequency of the waveform is twice that of the voltage about ie. 100 Hz in this case. Strictly it is not sinusoidal as it does not go negative, but we can say it is of sinusoidal shape with an offset. The offset is equivalent to the mean value of the power, calculated as follows

$$\text{Pmean} := \left(\frac{1}{\text{tot\_steps}}\right)\cdot \sum_{\text{mval}=1}^{\text{tot\_steps}} \text{Pinst}(\text{mval}\cdot\text{time\_step}) = 0.25 \qquad \text{W}$$

Alternatively we could have looked at that analytically as follows. the instantaneous power is given by:

$$P_{inst} = \frac{V_{inst}^{\;2}}{Load\_R}$$

$$= \frac{V_{peak}^{\;2}\sin^2\left(\frac{2.\pi.time\_s}{period}\right)}{Load\_R}$$

$$= \frac{V_{peak}^{\;2}}{2.Load\_R}\left(1-\cos\left(\frac{4.\pi.time\_s}{period}\right)\right)$$

That used the trigonometry identity

$$\sin^2\theta = \frac{1}{2}\;1-\cos\;2\theta$$

The long term average of either a sinewave or a cosine wave is zero, so the cosine term is replaced by zero giving.

$$P_{mean} = \frac{V_{peak}^{\;2}}{2.Load\_R}$$

Substituting 5 V for Vpeak and 50 Ohm for Load_R gives us the same result, 0.25 W

This is another way of defining the RMS voltage as the equivalent value of voltage that would have been used treating the power dissipation as one would with DC.

$$P_{mean} = \frac{{V_{RMS}}^2}{Load\_R}$$

So

$$V_{RMS} = \frac{V_{peak}}{\sqrt{2}}$$

The peak value of the power against time waveform is twice its mean value.

Now lets calculate the RMS value of the power waveform in exactly the same way as we did with RMS voltage but using the expression for instantaneous power instead.

$$\text{PRMS} := \sqrt{\left(\frac{1}{\text{tot\_steps}}\right)\cdot\sum_{\text{mval}=0}^{\text{tot\_steps}}(\text{Pinst}(\text{mval}\cdot\text{time\_step}))^2} = 0.306 \qquad \text{W}$$

So the RMS power differs from the mean (or average) power.

When people talk about RMS power what do they mean? A value for RMS power does exist but I think in most cases they really mean mean or average power and not RMS power. Mean power is the same as the heating effect of the power or the equivalent DC heating effect power. It is however derived from the RMS voltage (or RMS current) and of course the resistance.